

Name: _____

Score:

Part 1: _____ (20 possible)

Part 2: _____ (15 possible)

Part 3: _____ (15 possible)

Part 4: _____ (20 possible)

Part 5: _____ (15 possible)

Part 6: _____ (15 possible)

Total: _____ (100 total possible points)

Part 1: Binary Relations (20 points)

The five properties of binary relations that we are most interested in are: *reflexive*, *symmetric*, *transitive*, *irreflexive*, *antisymmetric*

For each of the following binary relations, state which of these properties they have (examples will be helpful for awarding partial credit):

The subset relation on sets.

The “has a common spoken language with” relation on people.

Part 2: Closure (15 points)

Find the *transitive closure* of the following relation over the set $\{a,b,c,d\}$

$\{(a,b),(b,d),(c,b),(d,a)\}$

Part 3: Composing Relations (15 points)

Write a suitable name for the following composition:

`isChildOf ° isChildOf`

`isSisterOf ° isParentOf`

Part 4: Partial Orders (20 points)

Let S be the set of strings consisting of the months of the year in English:

$$S = \{\text{j\text{a}n\text{u}a\text{r}\text{y}, \text{f\text{e}b\text{r}\text{u}a\text{r}\text{y}, \text{m\text{a}r\text{c}\text{h}, \text{a}\text{p}\text{r}\text{i}\text{l}, \text{m\text{a}\text{y}, \text{j}\text{u}\text{n}\text{e}, \text{j}\text{u}\text{l}\text{y}, \text{a}\text{u}\text{g}\text{u}\text{s}\text{t}, \text{s}\text{e}\text{p}\text{t}\text{e}\text{m}\text{b}\text{e}\text{r}, \text{o}\text{c}\text{t}\text{o}\text{b}\text{e}\text{r}, \text{n}\text{o}\text{v}\text{e}\text{m}\text{b}\text{e}\text{r}, \text{d}\text{e}\text{c}\text{e}\text{m}\text{b}\text{e}\text{r}\}$$

A partial order can be defined on S by letting $x < y$ iff $|x| < |y|$
where $|s|$ is the length of string s .

Draw a poset diagram of S over this relation:

Show a topologically sorted order of this set with respect to this relation:

Part 5: Equivalence Classes (15 points)

For each of the following functions f with domain N , describe the equivalence classes of the kernel relation of f .

$$f(x) = \text{floor}(x/2)$$

$$f(x) = x$$

Part 6: Partial Orders (15 points)

What properties of a binary relation are required for a binary relation to be a *partial order*?

Show how the poset $\langle \mathbb{N}, < \rangle$ has the properties you list above.