Sets

Class 3

1.2 Sets

- a set is an unordered collection of distinct "things" called elements
  1. no element is repeated
  2. there is no ordering in the collection
- sets are denoted by uppercase math letters; individual elements by lowercase math letters
- if x is an element of the set S, then we write $x \in S$ (LaTeX: `$x \in S$`)
- if a set has only a few elements, we can list them using curly braces: $S = \{x, y, z\}$ (LaTeX: `$S = \{x,y,z\}$`)
- we can use ellipsis to indicate a pattern $\mathbf{N} = \{0, 1, 2, \dots\}$ (LaTeX: `$\mathbf{N} = \{0,1,2,\dots\}$`)
- a set with no elements, the empty set, is denoted two ways: $S = \{\}$ or $S = \varnothing$ (LaTeX: `$S = \varnothing$`)

Infinite Sets

- some sets have a finite number of elements, some are infinite
- some infinite sets can be indicated with ellipsis
- some, such as the rational numbers, have too many for even that
- in this case, we use a description

$$\mathbf{Q} = \{x \mid x = \frac{m}{n}\text{ where } m,n \in \mathbf{Z}, n \neq 0\}$$

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\[
\mathbf{Q} = \{x \mid x = \frac{m}{n}\text{ where }
m,n \in \mathbf{Z}, n \neq 0\}
\]
```

Equal Sets and Subsets

- two sets are equal if they have exactly the same elements
$$\{x, y, z\} = \{y, x, z\}$$
- if every element in S is an element of T, then $S \subseteq T$ (LaTeX: `$S \subseteq T$`)
- $S \subseteq S$ is true for every set S (it is a tautology)
- $\varnothing \subseteq S$ also a tautology

Proper Subsets

- if $S \subseteq T$ and T has at least one element not in S, then S is a proper subset of T and we write $S \subset T$

here is another tautology

$$S \subset T \rightarrow S \subseteq T$$

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\[
S \subset T \rightarrow S \subseteq T
\]
```

and another

$$S \subseteq T \text{ and } T \subseteq S \rightarrow S = T$$

The Power Set

- every set S has a power set, denoted power(S), which is the set of all subsets of S

if $S = \{a, b, c\}$ then

$$\begin{aligned}
\text{power}(S) = \{&\{\}, \{a\}, \{b\}, \{c\},\\
&\{a,b\}, \{a,c\}, \{b,c\},\\
&\{a,b,c\}\}
\end{aligned}$$

```
\begin{align*}
\text{power}(S) = \{&\{\}, \{a\}, \{b\}, \{c\},\\
                    &\{a,b\}, \{a,c\}, \{b,c\},\\
                    &\{a,b,c\}\}\\
\end{align*}
```

Set Operations

- union: $S \cup T$ (`$S \cup T$`) is the set of all elements in S, or in T, or in both S and T: like logical OR
- intersection: $S \cap T$ (`$S \cap T$`) is the set of all elements that S and T have in common: like logical AND
- difference: $S - T$ is the set of elements of S that are not in T
- symmetric difference: $S \oplus T$ (`$S \oplus T$`) is the set of elements of in S but not in T, plus the elements in T but not in S: like exclusive-or
- complement: S' every set exists in some universe; S' is everything not in S: like logical NOT

Cardinality

- the number of elements in a set is its cardinality $|S|$ (`$|S|$`)
- there are two important counting rules with sets
  1. union rule, which can be extended to three or more sets

$$|A \cup B| = |A| + |B| - |A \cap B|$$

  2. difference rule

$$|A - B| = |A| - |A \cap B|$$

Properties

- section 1.2 is full of boxes that list properties and characteristics of set operations
- associativity, commutativity, etc.
- these are very important
- make sure you study them