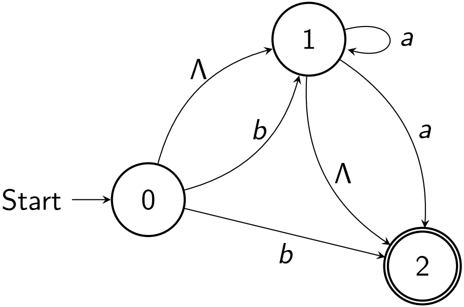
Finite Automata Esoterica (Section 11.3)

NFA to DFA

- The only thing we are going to cover from this section is how to transform an NFA into a DFA.
- First we define a λ-*closure* of a state s, denoted $\lambda(s)$, to be the set consisting of s together with all states that can be reached from s by traversing λ-edges. The λ-closure of a set S of states, denoted by $\lambda(S)$, is the union of the λ-closures of the states in S.

λ-closure examples

Given the following NFA as a graph and as a transition table:



	T_n	a	b	Λ
S	0	$\varnothing$	$\{1,2\}$	$\{1\}$
	1	$\{1,2\}$	$\varnothing$	$\{2\}$
F	2	$\varnothing$	$\varnothing$	$\varnothing$

Example λ-closures

Some same λ-closures for the previous NFA are:

- $\lambda(0) = \{0, 1, 2\}$
- $\lambda(1) = \{1, 2\}$
- $\lambda(2) = \{2\}$
- $\lambda(\varnothing) = \varnothing$
- $\lambda(\{1, 2\}) = \{1, 2\}$
- $\lambda(\{0, 1, 2\}) = \{0, 1, 2\}$

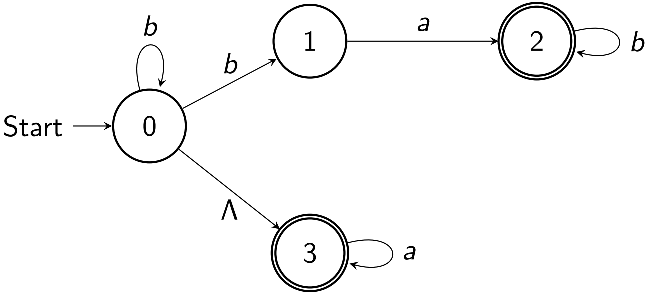
Transform an NFA to a DFA

Algorithm: Construct a DFA table T_D from an NFA table T_N as follows:

1. The start state of the DFA is $\lambda(s)$, where s is the start state of the NFA.
2. If $\{s_1, \ldots, s_n\}$ is a DFA state and $a \in A$, then:
   - $T_D(\{s_1, \ldots, s_n\}, a) = \lambda(T_N(s_1, a) \cup \ldots \cup T_N(s_n, a))$
3. A DFA state is final if one of its elements is an NFA final state.

Example

Given the following NFA:



	T_N	a	b	Λ
S	0	$\varnothing$	$\{0,1\}$	$\{3\}$
	1	$\{2\}$	$\varnothing$	$\varnothing$
F	2	$\varnothing$	$\{2\}$	$\varnothing$
F	3	$\{3\}$	$\varnothing$	$\varnothing$

Solution

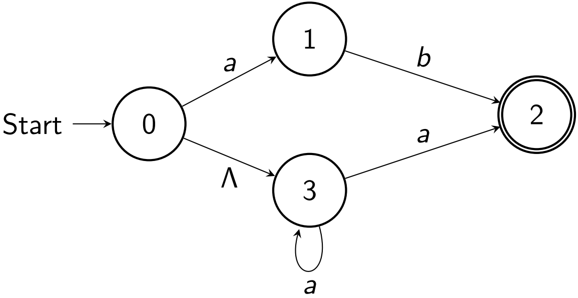
The algorithm constructs the following DFA transition table T_D, where we can simplify it by renaming the states as shown on the right:

	T_D	a	b
S,F	$\{0, 3\}$	$\{3\}$	$\{0, 1, 3\}$
F	$\{3\}$	$\{3\}$	$\varnothing$
F	$\{0, 1, 3\}$	$\{2, 3\}$	$\{0, 1, 3\}$
F	$\{2, 3\}$	$\{3\}$	$\{2\}$
F	$\{2\}$	$\varnothing$	$\{2\}$
	$\varnothing$	$\varnothing$	$\varnothing$

	T_D	a	b
S,F	0	1	2
F	1	1	5
F	2	3	2
F	3	1	4
F	4	5	4
	5	5	5

Another Example

Given the following NFA:



	T_N	a	b	Λ
S	0	$\{1\}$	$\emptyset$	$\{3\}$
	1	$\emptyset$	$\{2\}$	$\emptyset$
F	2	$\emptyset$	$\emptyset$	$\emptyset$
F	3	$\{2, 3\}$	$\emptyset$	$\emptyset$

Solution

The algorithm constructs the following DFA transition table T_D, where we can simplify it by renaming the states as shown on the right:

	T_D	a	b
S	$\{0,3\}$	$\{1,2,3\}$	$\varnothing$
F	$\{1,2,3\}$	$\{2,3\}$	$\{2\}$
F	$\{2,3\}$	$\{2,3\}$	$\varnothing$
F	$\{2\}$	$\varnothing$	$\varnothing$
	$\varnothing$	$\varnothing$	$\varnothing$

	T_D	a	b
S,F	0	1	4
F	1	2	3
F	2	2	4
F	3	4	4
	4	4	4

Rest of the section

The rest of this section is about minimizing the number of states in a DFA. We will not cover that topic, but I guess you should know that it is a thing that can be done.