Pushdown Automata (Section 11.6)
A pushdown automaton (PDA) is a finite automaton with a stack that has stack operations pop, push, and nop. PDAs always start with one designated symbol on the stack. A state transition depends on the input symbol and the top of the stack. The machine then performs a stack operation and enters the next state.

Representation can be graphical or with sets of 5-tuples. For example:

\[(i, b, C, \text{pop}, j)\]
Execution of previous slide

- **Execution:** If the machine is in state $i$ and the input letter is $b$ and $C$ is on the top of the stack, then pop the stack and enter state $j$. 
Nondeterminism in PDAs

- *Nondeterminism* can occur in two ways as shown:
A string $w$ is accepted by a PDA if there is a path from the start state to a final state such that the input symbols on the path edges concatenate to $w$. Otherwise, $w$ is rejected.
Example

A PDA to accept the language \( \{a^n b^n | n > 0\} \) as a graph and as a set of 5-tuples.

\[\begin{align*}
\text{Start} & \rightarrow 0 & \text{push}(a) \\
& \rightarrow 1 & \text{pop} \\
& \rightarrow 2 & \text{nop}
\end{align*}\]

- \((0, a, X, \text{push}(a), 0)\)
- \((0, a, a, \text{push}(a), 0)\)
- \((0, b, a, \text{pop}, 1)\)
- \((1, b, a, \text{pop}, 1)\)
- \((1, \Lambda, X, \text{nop}, 2)\)
How would you modify the machine to accept \( \{ a^n b^n \mid n \in \mathbb{N} \} \)?
Extension

How would you modify the machine to accept \( \{a^n b^n | n \in \mathbb{N}\} \)?

*Answer:* Add the instruction \((0, \Lambda, X, \text{nop}, 2)\).
Another Example

Find a PDA to accept the language $\{a^{2n} b^n | n \in \mathbb{N}\}$.
Context-Free Languages and PDAs

- **Theorem**: The context-free languages are exactly the languages accepted by PDAs.
- Proof could consist of showing how to transform a context-free grammar into a PDA, and showing how to transform a PDA into a context-free grammar. Both can be done, but we won't do them.
Nondeterminism Adds Power

- Nondeterministic PDAs are more powerful than deterministic PDAs.
- Consider the language of even palindromes over \{a,b\}. A context free grammar for this language is given by:
  - \( S \rightarrow \Lambda | aSa | bSb \)
- Any PDA to accept the language must make a nondeterministic decision to start comparing the second half of a string with the reverse of the first half.
Example

\[
\begin{align*}
\text{Start} & \quad 0 \quad 1 \quad 2 \\
\frac{a,X}{\text{push}(a)} & \quad \frac{a,?}{\text{push}(a)} \\
\frac{b,X}{\text{push}(b)} & \quad \frac{b,?}{\text{push}(b)} \\
\frac{\Lambda,?}{\text{nop}} & \\
\frac{\Lambda,X}{\text{nop}} &
\end{align*}
\]