Complexity Classes (Section 12.5)

Complexity Classes

- A complexity class is a set of problems related by how much time or space algorithms take to solve them. Complexity classes are usually defined for decision problems, which are problems that ask a question with a yes/no answer. Here are some fundamental complexity classes:
  - **The class P:** The set of problems that can be solved by deterministic algorithms with worst-case running time of polynomial order.
  - **The class NP:** The set of problems for which solutions can be verified by deterministic algorithms with worst-case running times of polynomial order. (The N stands for nondeterministic guess and the P stands for polynomial time to verify that the guess is a solution.)
  - **The class PSPACE:** The set of problems that can be solved by deterministic algorithms where the worst-case number of memory cells needed is of polynomial order.

Intractable Problems

- *Intractable problems:* are the set of problems that are not in P.
- *NP-Complete problems:* A problem is NP-Complete if it is in NP and is as hard as the hardest problems in NP (a category known at the NP-Hard problems). To show a problem is NP-Complete, you need to show that it is in NP (usually easy) and show that a known NP-Hard problem can be transformed (in polynomial time) to your problem.
- So, if a polynomial time solution is found for any NP-Complete problem, then that polynomial time solution could be used to solve any other NP-Complete problem in polynomial time.

P vs. NP

- Does $P = NP$? Does $P \neq NP$?
- No one definitely knows the answer to either question. The expectation of most is that $P \neq NP$ but it is clearly difficult to prove.
- One practical ramification of this for software developers is that if your boss asks you to write code to solve a known NP-Complete problem and you successfully produce a useful algorithm, then you should get a raise.
- In practice, approximations are used or the domain is limited to very small problems or other approaches are taken to avoid the NP-Complete problem. UPS still delivers packages, even though finding an optimal route to deliver any particular set of packages to particular addresses is NP-Complete.