Section 6.2 Practice Problems
Use known equivalences.

• Prove that $A \lor B \rightarrow C \equiv (A \rightarrow C) \land (B \rightarrow C)$.

• Prove that $(A \rightarrow B) \lor (\neg A \rightarrow B)$ is a tautology (i.e., show it is equivalent to true).

• Prove that $A \rightarrow B \equiv (A \land \neg B) \rightarrow \text{False}$. 
• Example: Use equivalences to show that
  \[ A \lor B \rightarrow C \equiv \neg (A \lor B) \lor C \]
• Proof:
  \[ A \lor B \rightarrow C \equiv \neg (A \lor B) \lor C \]
Practice example 1

Example: Use equivalences to show that
\[ A \lor B \rightarrow C \equiv \neg(A \lor B) \lor C \]

Proof:
\[ A \lor B \rightarrow C \equiv \neg(A \lor B) \lor C \]
\[ \equiv (\neg A \land \neg B) \lor C \]
Example: Use equivalences to show that
\[ A \lor B \rightarrow C \equiv \neg(A \lor B) \lor C \]

Proof:

\[
A \lor B \rightarrow C \quad \equiv \quad \neg(A \lor B) \lor C \\
\equiv \quad (\neg A \land \neg B) \lor C \\
\equiv \quad (\neg A \lor C) \land (\neg B \lor C)
\]
Practice example 1

• Example: Use equivalences to show that
  \( A \lor B \rightarrow C \equiv \neg (A \lor B) \lor C \)

• Proof:
  \[
  A \lor B \rightarrow C \equiv \neg (A \lor B) \lor C \\
  \equiv (\neg A \land \neg B) \lor C \\
  \equiv (\neg A \lor C) \land (\neg B \lor C) \\
  \equiv (A \rightarrow C) \land (B \rightarrow C)
  \]
Practice example 2

• Prove that \((A \rightarrow B) \lor (\neg A \rightarrow B)\) is a tautology (i.e., show it is equivalent to true).

• Proof:

\[
(A \rightarrow B) \lor (\neg A \rightarrow B) \equiv (\neg A \lor B) \lor (\neg \neg A \lor B)
\]
Practice example 2

• Prove that \((A \rightarrow B) \lor (\neg A \rightarrow B)\) is a tautology (i.e., show it is equivalent to true).

• Proof:

\[
(A \rightarrow B) \lor (\neg A \rightarrow B) \equiv (\neg A \lor B) \lor (\neg \neg A \lor B) \\
\equiv (A \lor \neg A) \lor (B \lor B) \\
\equiv \text{True} \lor \text{True} \\
\equiv \text{True}
\]
• Prove that \((A \rightarrow B) \lor (\neg A \rightarrow B)\) is a tautology (i.e., show it is equivalent to true).

• Proof:

\[
(A \rightarrow B) \lor (\neg A \rightarrow B) \equiv (\neg A \lor B) \lor (\neg \neg A \lor B) \\
\equiv (A \lor \neg A) \lor (B \lor B) \\
\equiv \text{True} \lor B
\]
Practice example 2

• Prove that \((A \rightarrow B) \lor (\neg A \rightarrow B)\) is a tautology (i.e., show it is equivalent to true).

• Proof:

\[
(A \rightarrow B) \lor (\neg A \rightarrow B) \equiv (\neg A \lor B) \lor (\neg \neg A \lor B) \\
\equiv (A \lor \neg A) \lor (B \lor B) \\
\equiv \text{True} \lor B \\
\equiv \text{True}
\]
Practice example 3

- Prove that $A \rightarrow B \equiv (A \land \neg B) \rightarrow \text{False}$.
- Proof:

  $$(A \land \neg B) \rightarrow \text{False} \equiv \neg(A \land \neg B) \lor \text{False}$$
Practice example 3

• Prove that $A \rightarrow B \equiv (A \land \neg B) \rightarrow \text{False}$.

• Proof:

\[
(A \land \neg B) \rightarrow \text{False} \equiv \neg (A \land \neg B) \lor \text{False} \\
\equiv (\neg A \lor B) \lor \text{False}
\]
Practice example 3

• Prove that \( A \rightarrow B \equiv (A \land \lnot B) \rightarrow \text{False} \).

• Proof:

\[
(A \land \lnot B) \rightarrow \text{False} \equiv \lnot (A \land \lnot B) \lor \text{False} \\
\equiv (\lnot A \lor B) \lor \text{False} \\
\equiv \lnot A \lor B
\]
Practice Quine’s Method

- Show that \((A \lor B \rightarrow C) \lor A \rightarrow (C \rightarrow B)\) is NOT a tautology.
Quine’s Method Example 1

Show that $W = (A \lor B \rightarrow C) \lor A \rightarrow (C \rightarrow B)$ is NOT a tautology.
Quine's Method Example 1

Show that $W = (A \lor B \rightarrow C) \lor A \rightarrow (C \rightarrow B)$ is NOT a tautology.

$W(A/\text{True } ) \equiv (\text{True} \lor B \rightarrow C) \lor \text{True} \rightarrow (C \rightarrow B)$

$\equiv (\text{True} \rightarrow (C \rightarrow B))$

$\equiv (C \rightarrow B)$

Let $X = (C \rightarrow B)$. Then we have:
Quine’s Method Example 1

Show that \( W = (A \lor B \rightarrow C) \lor A \rightarrow (C \rightarrow B) \) is NOT a tautology.

\[
W(A/\text{True}) \equiv (\text{True} \lor B \rightarrow C) \lor \text{True} \rightarrow (C \rightarrow B)
\equiv (\text{True} \rightarrow (C \rightarrow B))
\equiv (C \rightarrow B)
\]

Let \( X = (C \rightarrow B) \). Then we have:

\[
X(B/\text{True}) \equiv \text{True}
X(B/\text{False}) \equiv (C \rightarrow \text{False})
\equiv \neg C
\]
Quine’s Method Example 1

Show that \( W = (A \lor B \to C) \lor A \to (C \to B) \) is NOT a tautology.

\[
W(A/\text{True} ) \equiv ( \text{True} \lor B \to C) \lor \text{True} \to (C \to B) \\
\equiv (\text{True} \to (C \to B)) \\
\equiv (C \to B)
\]

Let \( X = (C \to B) \). Then we have:

\[
X(B/\text{True} ) \equiv \text{True} \\
X(B/\text{False} ) \equiv (C \to \text{False}) \\
\equiv \neg C
\]

Let \( Y = \neg C \). Then we have:

\[
Y(C/\text{True} ) \equiv \text{False} \\
Y(C/\text{False} ) \equiv \text{True}
\]

This is enough to show that it is a contingency.
Quine’s Method Example 1

Show that $W = (A \lor B \rightarrow C) \lor A \rightarrow (C \rightarrow B)$ is NOT a tautology.

\[
W(A/True) \equiv (True \lor B \rightarrow C) \lor True \rightarrow (C \rightarrow B) \\
\equiv (True \rightarrow (C \rightarrow B)) \\
\equiv (C \rightarrow B)
\]

Let $X = (C \rightarrow B)$. Then we have:

\[
X(B/True) \equiv True \\
X(B/False) \equiv (C \rightarrow False) \\
\equiv \neg C
\]

Let $Y = \neg C$. Then we have:

\[
Y(C/True) \equiv False \\
Y(C/False) \equiv True
\]

This is enough to show that it is a contingency.