Section 6.2 Practice Problems

Equivalence Practice Problems

Use known equivalences.

- Prove that $A \vee B \to C \equiv (A \to C) \wedge (B \to C)$.
- Prove that $(A \to B) \vee (\neg A \to B)$ is a tautology (i.e., show it is equivalent to true).
- Prove that $A \to B \equiv (A \wedge \neg B) \to \text{False}$.

Practice example 1

- Example: Use equivalences to show that $A \vee B \rightarrow C \equiv \neg(A \vee B) \vee C$
- Proof:

$$A \vee B \rightarrow C \quad \equiv \neg(A \vee B) \vee C$$

Practice example 1

- Example: Use equivalences to show that $A \vee B \rightarrow C \equiv \neg(A \vee B) \vee C$
- Proof:

$$\begin{aligned} A \vee B \rightarrow C \quad &\equiv \neg(A \vee B) \vee C \\ &\equiv (\neg A \wedge \neg B) \vee C \end{aligned}$$

Practice example 1

- Example: Use equivalences to show that $A \vee B \to C \equiv \neg(A \vee B) \vee C$
- Proof:

$$\begin{aligned} A \vee B \to C \quad &\equiv \neg(A \vee B) \vee C \\ &\equiv (\neg A \wedge \neg B) \vee C \\ &\equiv (\neg A \vee C) \wedge (\neg B \vee C) \end{aligned}$$

Practice example 1

- Example: Use equivalences to show that $A \vee B \to C \equiv \neg(A \vee B) \vee C$
- Proof:

$$\begin{aligned} A \vee B \to C &\equiv \neg(A \vee B) \vee C \\ &\equiv (\neg A \wedge \neg B) \vee C \\ &\equiv (\neg A \vee C) \wedge (\neg B \vee C) \\ &\equiv (A \to C) \wedge (B \to C) \end{aligned}$$

Practice example 2

- Prove that $(A \rightarrow B) \vee (\neg A \rightarrow B)$ is a tautology (i.e., show it is equivalent to true).
- Proof:

$$(A \rightarrow B) \vee (\neg A \rightarrow B) \quad \equiv (\neg A \vee B) \vee (\neg\neg A \vee B)$$

Practice example 2

- Prove that $(A \to B) \vee (\neg A \to B)$ is a tautology (i.e., show it is equivalent to true).
- Proof:

$$\begin{aligned} (A \to B) \vee (\neg A \to B) \quad &\equiv (\neg A \vee B) \vee (\neg\neg A \vee B) \\ &\equiv (A \vee \neg A) \vee (B \vee B) \end{aligned}$$

Practice example 2

- Prove that $(A \to B) \vee (\neg A \to B)$ is a tautology (i.e., show it is equivalent to true).
- Proof:

$$\begin{aligned}(A \to B) \vee (\neg A \to B) \quad &\equiv (\neg A \vee B) \vee (\neg\neg A \vee B)\\ &\equiv (A \vee \neg A) \vee (B \vee B)\\ &\equiv \text{True} \vee B\end{aligned}$$

Practice example 2

- Prove that $(A \to B) \vee (\neg A \to B)$ is a tautology (i.e., show it is equivalent to true).
- Proof:

$$
\begin{aligned}
(A \to B) \vee (\neg A \to B) &\equiv (\neg A \vee B) \vee (\neg\neg A \vee B) \\
&\equiv (A \vee \neg A) \vee (B \vee B) \\
&\equiv \text{True} \vee B \\
&\equiv \text{True}
\end{aligned}
$$

Practice example 3

- Prove that $A \to B \equiv (A \wedge \neg B) \to \text{False}$.
- Proof:

$$(A \wedge \neg B) \to \text{False} \quad \equiv \neg(A \wedge \neg B) \vee \text{False}$$

Practice example 3

- Prove that $A \to B \equiv (A \land \neg B) \to \text{False}$.
- Proof:

$$\begin{aligned}
(A \land \neg B) \to \text{False} \quad &\equiv \neg(A \land \neg B) \lor \text{False} \\
&\equiv (\neg A \lor B) \lor \text{False}
\end{aligned}$$

Practice example 3

- Prove that $A \to B \equiv (A \wedge \neg B) \to \text{False}$.
- Proof:

$$
\begin{aligned}
(A \wedge \neg B) \to \text{False} \quad & \equiv \neg(A \wedge \neg B) \vee \text{False} \\
& \equiv (\neg A \vee B) \vee \text{False} \\
& \equiv \neg A \ \vee B
\end{aligned}
$$

Practice Quine's Method

- Show that $(A \vee B \to C) \vee A \to (C \to B)$ is NOT a tautology.

Quine's Method Example 1

Show that $W = (A \vee B \to C) \vee A \to (C \to B)$ is NOT a tautology.

Quine's Method Example 1

Show that $W = (A \vee B \to C) \vee A \to (C \to B)$ is NOT a tautology.

$$\begin{aligned} W(A/\text{True}) &\equiv (\text{ True } \vee B \to C) \vee \text{ True } \to (C \to B) \\ &\equiv (\text{True} \to (C \to B)) \\ &\equiv (C \to B) \end{aligned}$$

Let $X = (C \to B)$. Then we have:

Quine's Method Example 1

Show that $W = (A \vee B \to C) \vee A \to (C \to B)$ is NOT a tautology.

$$\begin{aligned} W(A/\text{True}) &\equiv (\text{True} \vee B \to C) \vee \text{True} \to (C \to B) \\ &\equiv (\text{True} \to (C \to B)) \\ &\equiv (C \to B) \end{aligned}$$

Let $X = (C \to B)$. Then we have:

$$\begin{aligned} X(B/\text{True}) &\equiv \text{True} \\ X(B/\text{False}) &\equiv (C \to \text{False}) \\ &\equiv \neg C \end{aligned}$$

Quine's Method Example 1

Show that $W = (A \vee B \to C) \vee A \to (C \to B)$ is NOT a tautology.

$$\begin{aligned} W(A/\text{True}\ ) &\equiv (\ \text{True} \vee B \to C) \vee \text{ True} \to (C \to B) \\ &\equiv (\text{True} \to (C \to B) \\ &\equiv (C \to B) \end{aligned}$$

Let $X = (C \to B)$. Then we have:

$$\begin{aligned} X(B/\text{True}\ ) &\equiv \text{True} \\ X(B/\text{False}\ ) &\equiv (C \to \text{False}) \\ &\equiv \quad \neg C \end{aligned}$$

Let $Y = \neg C$. Then we have:

$$\begin{aligned} Y(C/\text{True}\ ) &\equiv \text{False} \\ Y(C/\text{False}\ ) &\equiv \text{True} \end{aligned}$$

Quine's Method Example 1

Show that $W = (A \vee B \to C) \vee A \to (C \to B)$ is NOT a tautology.

$$\begin{aligned} W(A/\text{True}) &\equiv (\text{ True } \vee B \to C) \vee \text{ True } \to (C \to B) \\ &\equiv (\text{True} \to (C \to B)) \\ &\equiv (C \to B) \end{aligned}$$

Let $X = (C \to B)$. Then we have:

$$\begin{aligned} X(B/\text{True}) &\equiv \text{True} \\ X(B/\text{False}) &\equiv (C \to \text{False}) \\ &\equiv \neg C \end{aligned}$$

Let $Y = \neg C$. Then we have:

$$\begin{aligned} Y(C/\text{True}) &\equiv \text{False} \\ Y(C/\text{False}) &\equiv \text{True} \end{aligned}$$

This is enough to show that it is a contingency.