Equality (Section 7.4)

Equality

A *first-order theory* is a first-order predicate calculus applied to the formalization of some subject (i.e., decide on some axioms to define basic properties of the objects). How can we describe the idea of equality of expressions in a first-order theory? One way to start is to say the following wff is valid: $\forall x\ (x = x)$.

Equality Properties

The $=$ symbol is just the infix form of an equality predicate, e.g., we could pick a predicate say e and let $e(x, y)$ mean x equals y. To reason with equality we need some properties.

Properties of Equality:

- Equality Axiom (EA): $t = t$
- Symmetric: $(t = u) \rightarrow (u = t)$
- Transitive: $(t = u) \wedge (u = v) \rightarrow (t = v)$
- Equals for Equals (EE):
  $(t = u) \rightarrow f(\ldots, t, \ldots) = f(\ldots, u, \ldots)$
- Extended Equals for Equals (EE): $(t = u) \wedge W(t) \rightarrow W(u)$.